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MEASUREMENT OF NON-PLANAR DIELECTRIC SAMPLES USING AN OPEN RESONATOR

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Abstract

A new technique for the measurement of the complex permittivity of dielectric samples having convex-concave surfaces using an open resonator is reported. The paper discusses the theory behind the new technique and describes measurements made at 11.6 GHz on perspex samples whose surfaces have radii of curvature as small as 330mm. The results obtained are in good agreement with those for the same material measured in flat sheet form.

Introduction

A problem of continuing interest is that of finding an accurate and convenient method of mapping the complex permittivity of a complexly shaped dielectric object, such as a missile radome, especially at high temperatures. Due to the particular geometry involved, measurements on convex-concave samples are thus mandatory. Most techniques employed previously for this type of measurement have been destructive in that samples of material have had to be removed from the object under investigation and carefully machined before being measured in a waveguide system. Further more, after the machining process the samples are normally flat. Open resonator techniques, however, appear to offer the advantages of non-destructive, in-situ measurement on non-planar samples.

The open resonator has been shown previously to provide a convenient and accurate tool for the measurement of the complex permittivity of dielectric materials in flat sheet form (1)-(6). Normally, measurements are made with the sample located centrally within the resonator so as to match the wavefronts approximately to the sample surfaces. Perturbation theory is then used to compensate for the deviation of the sample geometry from the ideal bi-convex geometry required. Since this deviation is usually small, experimental errors of less than 1% for relative permittivity and 10% for loss tangent are typical of those which have been reported for various flat samples.

In an earlier paper (7), we presented the results of a study into the measurement of bi-concave dielectric samples using an open resonator and examined the applicability of the perturbation theory in these cases. More recently, we have developed a new technique for the measurement of convex-concave dielectric samples and preliminary results were reported in (8). Further details of this new technique are reported here. In the new measurement configuration, convex-concave samples are measured at off-centre positions inside the open resonator where the wavefronts are curved and therefore similar in form to the sample surfaces.

Theory

An open resonator formed by a pair of identical spherical mirrors, as shown in Fig. 1, supports a complete and orthogonal set of resonance modes. In practice, however, due to the diffraction losses caused by limited mirror aperture, only the fundamental and a few low-loss higher order modes actually exist. In the analysis which follows, only the fundamental mode of resonance is considered. This can be described in terms of a Gaussian beam which propagates between the mirrors to form a standing wave pattern. The beam has its minimum width at the centre of the resonator and the radius of curvature of the wavefronts decreases from infinity at the centre of the resonator to that of the mirrors at their surfaces. As the radius of curvature of the wavefronts varies along the axis of the resonator, accurate measurements on convex-concave samples should be possible by placing them nearer to one of the mirrors, so that the radii of curvature of the sample surfaces are similar to those of the wavefronts. The new measurement configuration is shown in Fig. 2. As the earlier theories only allow samples to be measured at the centre of the resonator, a new theory was needed to analyse our results. When a convex-concave sample with surfaces having the same radii of curvature as the wavefronts is placed inside the resonator, the system can be retuned by changing the resonant frequency. The changes in resonant frequency and Q factor are related to the relative permittivity and loss tangent and these can be calculated using the theory discussed below. Since most of the energy is concentrated near the longitudinal axis of the beam, as long as the sample is large enough to intercept the latter, the transverse dimensions of the sample are not critical.

When, for example, a sample is located nearer to the right hand mirror, as shown in Fig. 2, the system can be envisaged as being comprised of four regions, separated by three constant phase surfaces. These are the plane surface SA, where the beamwidth is a minimum, and the two air-dielectric interfaces SB and SC. At resonance, the wave impedances on both sides of the two curved air-dielectric interfaces can be equated if these correspond to constant phase surfaces; hence the radii of curvature of the sample surfaces are required to match those of the wavefronts. The beamwidth should also be identical on both sides of the air-dielectric interfaces. By applying these boundary conditions, two transcendental equations can be derived and used to determine the relative permittivity of the sample. Equating wave impedances at the interface SB.

$$jZ_o\tan(kD_b-Q_2(D_b)+kD_e-Q_2(D_e))$$
$$= jZ_1\tan(nkD_b-Q_3(D_b) + \beta) \quad \text{.... (1)}$$

whereas at interface SC

$$jZ_1\tan(nkD_t-Q_3(D_t) + \beta)$$
$$= jZ_o\tan(kD_t-Q_4(D_t)-kD_1+Q_4(D_1)) \quad \text{....(2)}$$

where n = refractive index of the dielectric,
k = free space phase constant,
Z_o = free space wave impedance,
Z_1 = wave impedance of the dielectric,
Q_2 = additional phase shift in region 2,
Q_3 = additional phase shift in region 3,
Q_4 = additional phase shift in region 4,
β = a phase constant determined by the specimen position,
D_b = distance from SA to SB,
D_t = distance from SA to SC,
D_1 = distance from SA to right hand mirror,
D_e = distance from SA to left hand mirror.

When the relative permittivity ε_r has been determined by $\varepsilon_r = n^2$, the loss tangent of the specimen can be obtained by calculating the difference between the energy losses of the unloaded and the loaded resonator. The following equation is used in this case,

$$\tan\delta = \left(\frac{1}{Q_\ell} - \frac{1}{Q_e}\right) \left(\int_{V_1} |E_1|^2 dV + \int_{V_2} |E_2|^2 dV + \varepsilon_r \int_{V_3} |E_3|^2 dV + \int_{V_4} |E_4|^2 dV\right) \Big/ \left(\varepsilon_r \int_{V_3} |E_3|^2 dV\right) \quad \text{....(3)}$$

where Q_e = unloaded Q factor,
Q_ℓ = loaded Q factor,
E_1 = the electric field in region 1,
E_2 = the electric field in region 2,
E_3 = the electric field in region 3,
E_4 = the electric field in region 4,
V_1 = whole volume of region 1,
V_2 = whole volume of region 2,
V_3 = whole volume of region 3,
V_4 = whole volume of region 4.

When the radii of curvature of the sample surfaces are different from those of the wavefronts, matching of the wave impedances at the air-dielectric interfaces is impossible. However, small deviations of sample geometry from the ideal can be compensated for by using perturbation theory (1).

Experimental Study

As a test of the new theory, measurements were made at 11.6 GHz on convex-concave perspex samples with radii of curvature as small as 330mm. The results are shown in Table 1, together with the corresponding results for the same material measured in flat sheet form, using the same system. As can be seen, the results for the two cases appear consistent and are in good agreement. The slight differences between the results for samples 1-5 and 6-8 is attributed to the fact that each group of samples was cut from different sheets of material.

Experimental errors appear to be due mainly to uncertainties in the measurement of sample geometry, sample position, resonant frequency and Q factor. When making measurements, the position of the sample inside the resonator needs to be known accurately, and this has proved to be a source of difficulty. However, by adjusting the position of the sample to produce either a maximum or a minimum shift in resonant frequency, only a rough estimation of the sample position is required. The actual position can then be calculated using a technique which seeks the optimum value of the relative permittivity, expressed as either a maximum or a minimum calculated value, as the position of the sample is allowed to vary slightly about the estimated value. By using this technique, the uncertainty in sample position was kept less than ± 0.5mm. By changing the assumed sample position by ± 0.5mm, variations in relative permittivity and loss tangent for all samples were calculated to be less than 1.5% and 15% respectively. Variations for samples 1 and 8 are shown in Table 2.

In our repeated thickness measurements (thickness at the centre of the samples), a measurement accuracy of ± 0.03mm was observed. By changing the assumed thickness in our analysis, variations in relative permittivity were less than 0.2% when samples were measured in high field regions of the resonator. Variations in relative permittivity were larger when samples were measured in low field regions in the resonator, where the relative field intensity at the air-dielectric interfaces was high. In this case, the variations were 1.5%. Variations in loss tangent for samples measured in both high and low field intensity regions were small. Table 3 shows the variations in relative permittivity and loss tangent due to assumed variations in thickness.

Perturbation theory compensation has been applied to the results to account for the deviations from the ideal geometry of the convex-concave samples, but since these were designed to match approximately with the wavefronts, the compensation is small. When, however, the sample geometry is very different from the ideal, the errors are expected to be large and dominated by those introduced by the use of perturbation theory. However, these errors can still be minimised, as reported in (7), by locating the sample in a high field region of the resonator standing wave pattern, whilst leaving the air-dielectric interfaces in relatively low field regions. Hence refraction of the electric field can be minimised together with the errors. Alternatively by using mirrors whose radii of curvature are similar to those of the sample surfaces, wavefronts with similar radii of curvature can be produced to match the sample geometry.

Errors in relative permittivity and loss tangent arising from uncertainties in the measured resonant frequency and Q factor are typically within 1% and 5% respectively. After taking the above mentioned factors into account, the uncertainties in the relative permittivity and loss tangent are expected to be within 3% and 20% respectively. However, in our results, errors up to 2% and 40% were obtained for relative permittivity and loss tangent respectively. Errors in loss tangent are very large indeed. However, they appear in a predictable manner, in that results obtained for samples measured in similar field intensity regions appear to be consistent, but a relatively large difference occurs between those measured in high and low field regions. As this effect is more serious when the beamwidth is large, it is thought to be due to excessive scattering losses at the air-dielectric interfaces which have yet to be taken into account.

Discussion

The new measurement technique has certain advantages over that employed for flat sheet samples. As the sample is not required to be positioned at a particular point within the resonator, the resonant frequency can be varied by moving the sample along the longitudinal axis. Measurement of the complex permittivity over a range of frequencies should therefore be possible although the frequency range may not be extensive. However, by measuring the sample at different positions along the longitudinal axis using several different axial order modes, the frequency range can be greatly extended.

Since the new technique is capable of measuring a sample positioned close to one of the mirrors, a symmetrical resonator is not always necessary. This is a particularly useful feature when access around the object under investigation is limited. Thus a non-symmetrical resonator formed from a flat mirror and a concave mirror, or an even more compact configuration of a convex mirror and a concave mirror could be used.

The new technique does have one disadvantage, however. Normally, for the measurement of flat samples, only the thickness of the latter need be known accurately. When using the new configuration, however, more details of the sample geometry are required; thus additional errors can arise.

Conclusions

A new technique for the measurement of convex-concave dielectric samples using an open resonator has been developed which involves positioning a sample such that its surfaces coincide approximately with particular wavefronts within the resonator. Measurements have been made at 11.6GHz on perspex samples with radii of curvature as small as 330mm. The results obtained appear consistent and are in good agreement with those for the same material measured in flat sheet form.

When the sample surfaces are approximately coincident with the wavefronts, experimental errors are due mainly to uncertainties in the sample geometry, sample position, resonant frequency and Q factor. Experimental errors within 2% for relative permittivity and 40% for loss tangent have been obtained. Errors in loss tangent are larger than expected and this is thought to be due to scattering at the air-dielectric interfaces which has yet to be taken into account.

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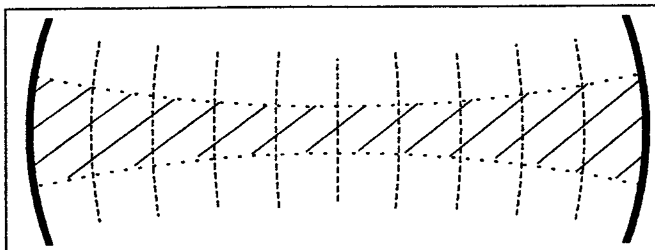

Fig.1 An open resonator formed by a pair of spherical mirrors.

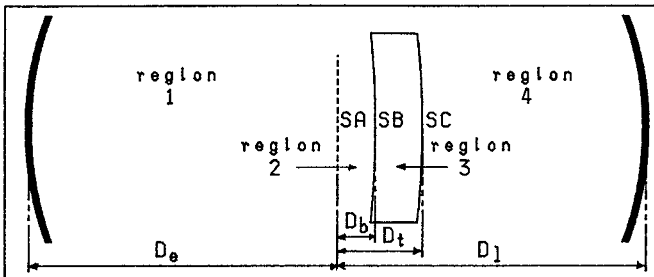

Fig.2 New measurement configuration.

Table 1 - Measurement results for flat and convex-concave samples

Frequency : 11.59 GHz
Material : Perspex
Radius of curvature of mirror : 330mm

Sample number	radius of curvature of surface SB	radius of curvature of surface SC	relative permittivity	loss tangent	field intensity	relative permittivity	loss tangent	field intensity
1	5.603m	3.787m	2.624	0.0069	high	2.613	0.0066	low
2	2.497m	2.062m	2.625	0.0069	high	2.601	0.0066	low
3	1.402m	1.257m	2.627	0.0071	high	2.591	0.0067	low
4	0.874m	0.823m	2.628	0.0073	high	2.622	0.0068	low
5	0.648m	0.625m	2.630	0.0079	high	2.625	0.0072	low
6	0.488m	0.478m	2.589	0.0052	high	2.601	0.0061	low
7	0.386m	0.384m	2.580	0.0058	high	2.607	0.0070	low
8	0.330m	0.330m	2.586	0.0061	high	2.596	0.0084	low
9	* flat sample *		2.628	0.0069	high	2.613	0.0066	low

Table 2 - Variations in relative permittivity and loss tangent due to uncertainty in sample position

Sample number	sample position	relative permittivity	loss tangent	field intensity	relative permittivity	loss tangent	field intensity
1	De-0.5mm	2.657	0.0070	high	2.583	0.0068	low
1	De	2.628	0.0069	high	2.613	0.0066	low
1	De+0.5mm	2.651	0.0070	high	2.588	0.0067	low
8	De-0.5mm	2.613	0.0061	high	2.577	0.0095	low
8	De	2.587	0.0064	high	2.604	0.0088	low
8	De+0.5mm	2.613	0.0069	high	2.574	0.0083	low

Table 3 - Variations in relative permittivity and loss tangent due to uncertainty in sample thickness

Sample number	sample thickness	relative permittivity	loss tangent	field intensity	relative permittivity	loss tangent	field intensity
8	5.84mm	2.590	0.0065	high	2.644	0.0085	low
8	5.87mm	2.587	0.0065	high	2.623	0.0084	low
8	5.90mm	2.585	0.0065	high	2.603	0.0084	low